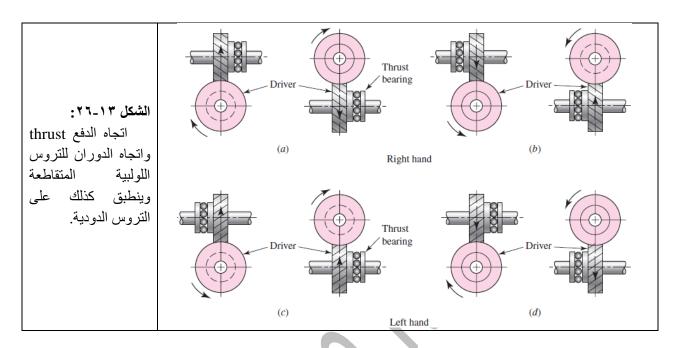
ملاحظة مهمة (تحديد اتجاه الدوران):

في حالة التروس العدلة spur واللولبية المتوازية helical يتم تحديد اتجاه الدوران باستخدام قاعدة اليد اليمنى ويكون الدوران موجبا إذا كان بعكس عقارب الساعة.

من الصعب تحديد اتجاه الدوران للتروس الدودية worm واللولبية المتقاطعة crossed helical لذا يستخدم الشكل ٣٦-١٣ المبين في ادناه، في تحديد اتجاه الدوران في مثل هذه الحالات.



ىثال:

في الشكل المجاور إذا كان العمود a يدور بسرعة 600 rpm جد اتجاه وسرعة الدوران للعمود d.

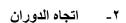
الحل:

١ - سرعة الدوران

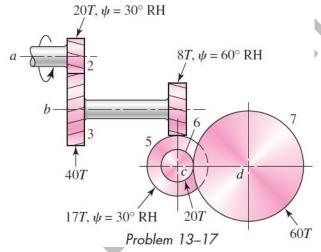
 $e = \frac{n_L}{n_F} = \frac{\text{product of driving tooth numbers}}{\text{product of driving tooth numbers}}$

$$e = \frac{n_2}{n_3} \frac{n_4}{n_5} \frac{n_6}{n_7} = \frac{20}{40} \frac{8}{17} \frac{20}{60} = \frac{4}{51}$$

$$n_L = e \, n_F = \frac{4}{51}(600) = 47.06 \, rpm$$



باتجاه عقارب الساعة (بالاستفادة من الشكل ١٦-١٣ الحالة b



تحليل القوة – التروس العدلة Spur Gearing تحليل القوة – التروس العدلة

يبين الشكل (17 - 17) ترسا صغيرا pinion تم تثبيته على العمود a الذي يدور باتجاه عقرب الساعة بسرعة u وشعاعية الترس المنقاد u فيدور بسرعة u يمكن تحليل قوة رد الفعل بين اسنان الترسين الى مركبتين مماسية u وشعاعية u

$$W_t = F_{23}^t \qquad \dots \dots (a)$$

$$T = \frac{d}{2}W_t \qquad \dots \dots (b)$$

$$W_t = \frac{2T}{d} \qquad \dots \dots (c)$$

$$W_r = W_t \tan \phi \qquad \dots \dots (d)$$

The power H transmitted through a rotating gear can be obtained of the product of torque T and ω , $H = T\omega$

$$H = \left(\frac{W_t d}{2}\right) \left(\frac{2\pi n}{60}\right) = \left(\frac{\pi dn}{60}W_t\right)$$

$$W_t = \frac{60000H}{\pi dn} \qquad (13 - 36)$$

 $W_t = transmitted load, kN$

H = power, kW

d = gear diameter, mm

n = speed, rev/min

Example 13.7

Pinion 2 in Figure 13–30a runs at 1750 rev/min and transmits 2.5 kW to idler gear 3. The teeth are cut on the 20° full-depth system and have a module of m = 2.5 mm. Draw a free-body diagram of gear 3 and show all the forces that act upon it.

Solution

The pitch diameters of gears 2 and 3 are

$$d_2 = N_2 m = 20(2.5) = 50 \text{ mm}$$

$$d_3 = N_3 m = 50(2.5) = 125 \text{ mm}$$

From Equation (13–36) we find the transmitted load to be

$$Wt = \frac{60000H}{\pi d_2 n} = \frac{60000(2.5)}{\pi (50)(1750)} = 0.546 \text{ kN}$$

Thus, the tangential force of gear 2 on gear 3 is $F_{23}^t = 0.546$ kN, as shown in Figure 13–30*b*. Therefore

$$F_{23}^r = F_{23}^t \tan 20^\circ = (0.546) \tan 20^\circ$$

= 0.199 kN

and so

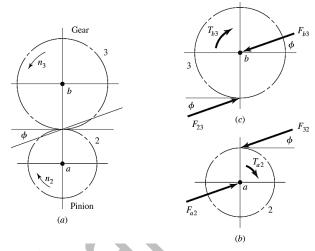


Figure 13–32 Free-body diagrams of two gears

$$F_{23} = F_{23}^t \cos 20^\circ = 0.546 \cos 20^\circ$$

= 0.581 kN

Since gear 3 is an idler, it transmits no power (torque) to its shaft, and so the tangential reaction of gear 4

on gear 3 is also equal to W_t . Therefore

$$F_{43}^t = 0.546 \text{ kN}$$
 $F_{43}^r = 0.199 \text{ kN}$ F_{43}
= 0.581 kN

and the directions are shown in Figure 13–30*b*.

The shaft reactions in the x and y directions are

$$F_{b3}^{x} = -(F_{23}^{t} + F_{43}^{r}) = -(-0.546 + 0.199)$$
$$- 0.347 \text{ kN}$$

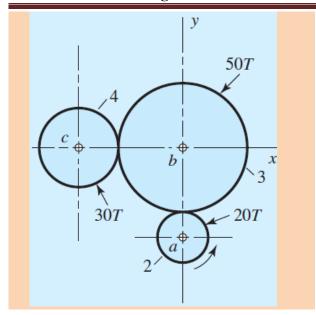
$$F_{b3}^{y} = -(F_{23}^{r} + F_{43}^{t}) = -(0.199 - 0.546)$$

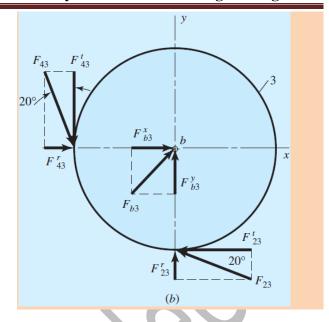
= 0.347 kN

The resultant shaft reaction is

$$F_3^b = \sqrt{(0.347)^2 + (0.347)^2} = 0.491 \text{ kN}$$

These are shown on the figure.





Force Analysis—Parallel Helical Gearing

The helix angle is the same on each gear, but one gear must have a right-hand helix and the other a left-hand helix. The shape of the tooth is an involute helicoid and is illustrated in Fig. 13–21

The initial contact of helical-gear teeth is a point that extends into a line as the teeth come into more engagement

Helical gears subject the shaft bearings to both radial and thrust loads.

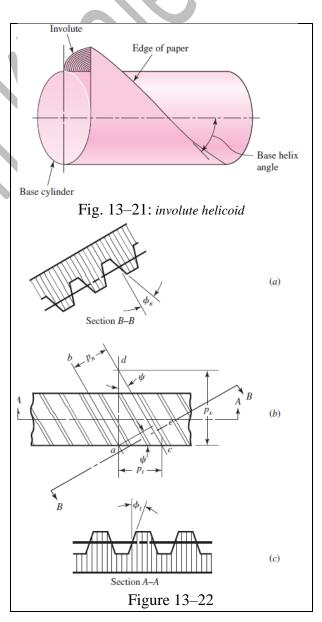
- Figure 13–22 represents a portion of the top view of a helical rack.
- Lines *ab* and *cd* are the centerlines of two adjacent helical teeth taken on the same pitch plane.
- The angle ψ is the *helix angle*.
- The distance ac is the p_t in the plane of rotation (usually called the $circular\ pitch$).
- The distance ae is the normal circular pitch p_n

$$p_n = p_t \cos \psi \quad \dots \dots (13 - 16)$$
$$m_n = m_t \cos \psi$$

• The distance ad is called the axial pitch p_x

$$p_x = \frac{p_t}{\tan \psi} \dots \dots (13 - 17)$$

The pressure angle ϕ_n in the normal direction is different from the pressure angle ϕ_n in the direction of rotation, because of the angularity of the teeth. These angles are related by the equation



$$\cos \psi = \frac{\tan \phi_n}{\tan \phi_t} \dots \dots (13 - 19)$$

For force analysis in helical gears, three components of the total (normal) tooth force W, are:

$$W_r = W \sin \phi_n$$

$$W_t = W \cos \phi_n \cos \psi$$

$$W_a = W \cos \phi_n \sin \psi$$
... ... (13 – 39)

W = total force

Wr = radial component

Wt =tangential component

Wa = axial component, (thrust load)

Usually, W_t is given and the other forces are desired. So:

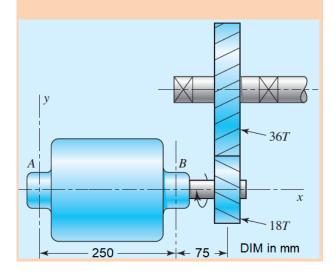
$$W_r = W_t \tan \phi_t$$

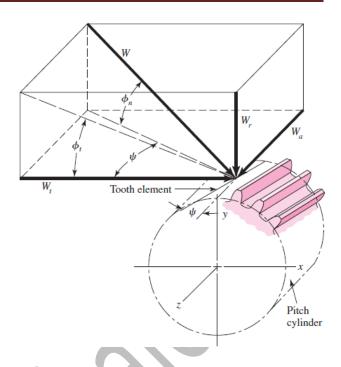
$$W_a = W_t \tan \psi$$

$$W = \frac{W_t}{\cos \phi_n \cos \psi}$$
.....(13 - 40)

Example 13-9

In Fig. 13–38 a 0.75 kW electric motor runs at 1800 rev/min in the clockwise direction, as viewed from the positive x axis. Keyed to the motor shaft is an 18-tooth helical pinion having a normal pressure angle of 20° , a helix angle of 30° , and a normal module m=3.0. The hand of the helix is shown in the figure. Make a three-dimensional sketch of the motor shaft and pinion, and show the forces acting on the pinion and the bearing reactions at A and B. The thrust should be taken out at A.





Solution:

From Equation (13–19) we find

$$\phi_t = \tan^{-1} \frac{\tan \phi_n}{\cos \psi} = \tan^{-1} \frac{\tan 20^\circ}{\cos 30^\circ} = 22.8^\circ$$

 $m_t = m_n/\cos\psi = 3/\cos 30^\circ = 3.46 \text{ mm}.$ $d_p = 18(3.46) = 62.3 \text{ mm}.$

The pitch-line velocity is

$$V = \frac{\pi dn}{60} = \frac{\pi (62.3)(1800)}{60} = 5871.6 \text{ mm/s}$$
$$= 5.87 \text{ m/s}$$

The transmitted load is

$$W_t = \frac{H}{V} = \frac{750}{5.87} = 128 \, N$$

From Equation (13–40) we find

 $W_r = W_t \tan \phi_t = 128 \tan 22.8^\circ = 54 N$

 $W_a = W_t \tan \psi = 128 \tan 30^\circ = 74 N$

$$W = \frac{W_t}{\cos \phi_n \cos \psi} = \frac{128}{\cos 20^\circ \cos 30^\circ}$$
$$= 157 N$$

These three forces,

 $W_r = 54 N$ in the -y direction

 $W_a = 74 N$ in the -x direction,

 $W_t = 128 N$ in the +z direction, are shown acting at point C in Figure 13–35.

Gears

assume bearing reactions at A and B as shown. Then $F_{xA} = W_a = 74 N$.

Taking moments about the z axis,

$$-(54.0)(325) + (74.0)(31.15) + 250F_{yB}$$
$$= 0$$

$$F_{vB} = 61$$

Summing forces in the y direction then gives;

$$F_{vA} = 7 N$$

Taking moments about the y axis, next

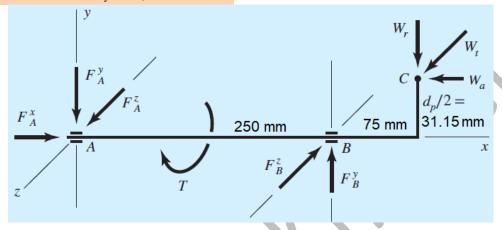
$$250F_{ZB} - 128(325) = 0$$
$$F_{ZB} = 166 N$$

Summing forces in the z direction and solving gives

$$F_{zA} = 38 N$$

. Also, the torque is

$$T = \frac{W_t d_p}{2} = 128(31.15) = 3982 \text{ N} \cdot \text{m}.$$



EXAMPLE 13–2 A stock helical gear has a normal pressure angle of 20°, a helix angle of 25°, and a transverse module of 6 mm, and has 18 teeth. Find:

- (a) The pitch diameter
- (b) The transverse, the normal, and the axial pitches
 - (c) The normal module
 - (d) The transverse pressure angle

Solution:

- (a) $d = Nm_t = 18(5) = 90 mm$
- (b) $p_t = \pi m_t = \pi(5) = 15.71 \, mm$

 $p_n = p_t \cos \psi = 15.71 \cos 25^\circ = 14.24 \, mm$

$$p_x = \frac{p_t}{\tan \psi} = \frac{15.71}{\tan 45^{\circ}} = 15.71 \ mm$$

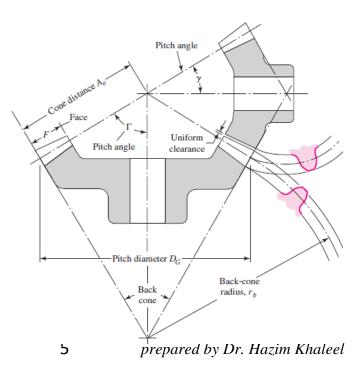
- (c) $m_n = m_t \cos \psi = 5 \cos 45^\circ = 3.54 \, mm$
- (d) $\phi_t = \tan^{-1}(\tan \phi_n / \cos \psi) =$

 $\tan^{-1}(\tan 45^{\circ}/\cos 25^{\circ}) = 21.88^{\circ}$

التروس المخروطية Bevel Gears - التروس المخروطية يتم تعريف زاوية الخطوة من خلال المخروطين الذين يلتقيان في البورة كما في الشكل، يمكن ايجاد الزوايا من عدد اسنان الترسين بالعلاقتين الاتية:

$$\tan \gamma = \frac{N_P}{N_G}
\tan \Gamma = \frac{N_G}{N_P} \qquad \dots \dots (13 - 14)$$

Pinion نصف زاوية المخروط للترس الصغير γ: نصف زاوية المخروط للترس الكبير Γ: نصف زاوية المخروط للترس الكبير

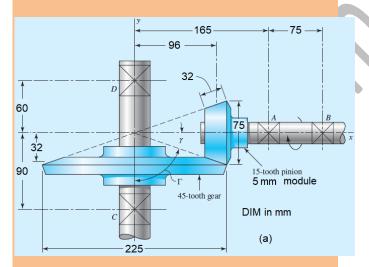


Force Analysis—Bevel Gearing

$$W_t = \frac{T}{r_{av}} \dots \dots (13 - 37)$$

 $r_{\rm av}$ is the pitch radius

Example The bevel pinion shown in Fig. rotates at 600 rev/min in the direction shown and transmits 3.75 kW to the gear. The mounting distances the location of all bearings, and the average radii of the pitch circles of the pinion and gear are shown in the figure. For simplicity, the teeth have been replaced by the pitch cones. Bearings A and C should take the thrust loads. Find bearing forces on the gear shaft.



Solution The pitch angles are

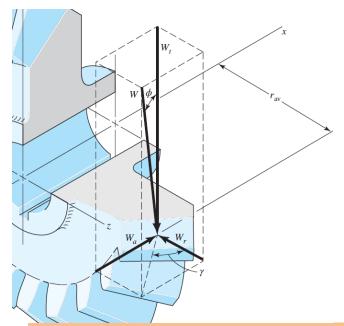
$$\gamma = \tan^{-1}\left(\frac{75}{225}\right) = 18.4^{\circ}$$

$$\Gamma = \tan^{-1}\left(\frac{225}{75}\right) = 71.6^{\circ}$$

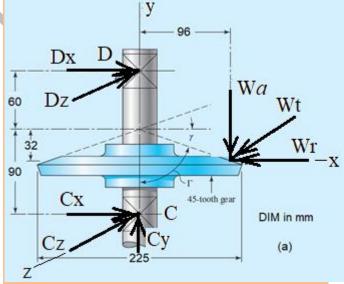
$$V = \frac{\pi d_p n}{60} = \frac{\pi(2r_p)n}{60} = \frac{\pi(2 \times 32)(600)}{60}$$

$$= 2011 \frac{\text{mm}}{\text{s}} = 2 \frac{\text{m}}{\text{s}}$$

$$W_t = \frac{H}{V} = \frac{3750}{2} = 1875 \text{ N}$$



 $W_{Pr} = W_{Ga} = W_t \tan \phi \cos \gamma$ = 1875 tan 20° cos 18.4° = 647.6 N $W_{Pa} = W_{Gr} = W_t \tan \phi \sin \gamma$ = 1875 tan 20° sin 18.4° = 215.4 N



$$C_y = W_a = 647.6 N$$

 $\Sigma M_{D_Z} = 0$
 $150C_x - 92W_r - 96W_a = 0$
 $150C_x - 92(215.4) - 96(647.6) = 0$
 $C_x = 546.6 N$
 $\Sigma M_{D_x} = 0$
 $150C_z - 92W_t = 0$
 $150C_z - 92(1875) = 0$

 $C_z = 1150 \text{ N}$

$$\Sigma F_{x}=0$$

$$-W_r + D_x + C_x = 0$$

$$-215.4 + D_x + 546.6 = \mathbf{0}$$

$$D_x = -331.2 N$$

$$\Sigma F_z = 0$$
 $W_t + D_z + C_z = 0$
 $1875 + D_z + 1150 = 0$
 $D_z = 725 N$

13-11 Worm Gears

$$\lambda = \psi_G$$
 for 90° shafts

$$\lambda + \psi_W = 90^{\circ}$$

κ: زاوية تقدم السن معادلة (١٣-٢٨)

زاوية الحلزنة للدودة: ψ_W

الخطوة القطرية للترس تساوي الخطة المحورية للترس الدودي إذا كانت الاعمدة متعامدة أي:

$$p_x = p_t = \frac{\pi d_G}{N_G}$$

قطر الترس:

$$d_G = \frac{N_G p_t}{\pi} \dots \dots (13 - 25)$$

يجب اختيار قطر الترس الدودي ليقع ضمن المدى الاتى من اجل تحقيق أفضل نقل للقدرة

$$\frac{C^{0.875}}{3.0} \le d_W \le \frac{C^{0.875}}{3.0} \dots \dots (13-26)$$

where *C* is the center distance.

$$C=\frac{d_G+d_W}{2}$$

The *lead L* is equal to:

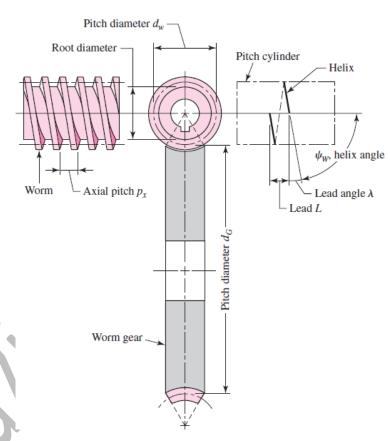
$$L = p_r N_W \dots (13 - 27)$$

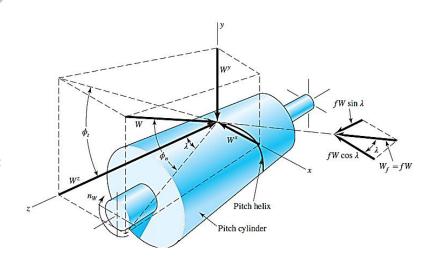
The *lead angle* λ of the worm is:

$$\tan \lambda = \frac{L}{\pi d_W} \dots \dots (13 - 28)$$

Note that

$$\frac{\omega_G}{\omega_W} = \frac{N_W}{N_G} = \frac{L}{\pi d_G}$$





Force Analysis (Without Friction)

Force Analysis (With Friction)

الحركة النسبية بين الدودة والترس تمثل حركة انز لاقية خالصة، لذا فان الاحتكاك يلعب دورا مهما في أداء التروس الدودية قوة الاحتكاك بين الترس والدودة W_1 :

$$W_f = \mu W$$
 with two components $W_{fx} = \mu W \cos \lambda$ $W_{fz} = \mu W \sin \lambda$

$$W_f = \mu W = \frac{\mu W_{Gt}}{(\mu \sin \lambda - \cos \phi_n \cos \lambda)} \dots \dots (13 - 44)$$

the worm tangential force, W_{Wt} :

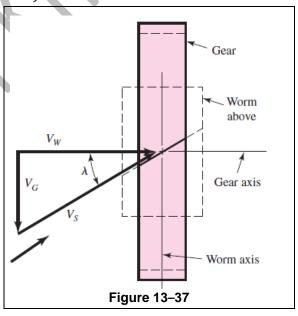
$$W_{Wt} = W_{Gt} \frac{\cos \phi_n \cos \lambda + \mu \sin \lambda}{(\mu \sin \lambda - \cos \phi_n \cos \lambda)} \dots \dots (13 - 45)$$

Efficiency η can be defined by using the equation:

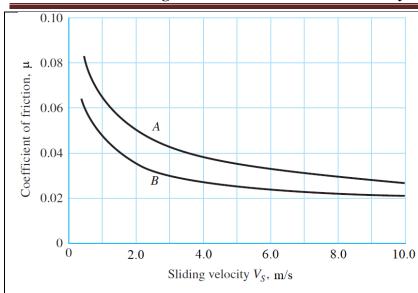
$$\eta = \frac{W_{Wt}(\text{Without Friction})}{W_{Wt}(\text{With Friction})} \dots (a)$$

$$\eta = \frac{\cos \phi_n - \mu \tan \lambda}{\cos \phi_n + \mu \cot \lambda} \dots (13 - 46)$$

Many experiments have shown that the coefficient of friction is dependent on the relative or sliding velocity. In Fig. 13–41, V_G is the pitch-line velocity of the gear and V_W the pitch-line velocity of the worm. Vectorially, $V_W = V_G + V_s$; the sliding velocity is:



$$V_S = \frac{V_W}{\cos \lambda} \dots \dots (13 - 47)$$



Representative values of the coefficient of friction for worm gearing. These values are based on good lubrication. Use curve *B* for high-quality materials, such as a case-hardened steel worm mating with a phosphorbronze gear. Use curve *A* when more friction is expected, as with a cast-iron worm mating with a cast-iron worm gear

Figure 13-42

Example: A 2-tooth right-hand worm transmits 0.75 kW at 1200 rev/min to a 30-tooth worm gear. The worm has an axial pitch of 13 mm, a pitch diameter of 50 mm, and a face width of 63 mm. The face width of the gear is 25 mm and normal pressure angle is 14.5°. The materials coefficient of friction is 0.03.

- (a) Find the center distance, the lead, and the lead angle.
- (b) Figure 13–39 is a drawing of the gearset oriented with respect to the coordinate system described earlier in this section; the gear is supported by bearings at A and B. Find the forces exerted by the bearings against the worm-gear shaft, and the output torque.

Solution: $N_W = 2, H = 750 W, n_W = 1200 rpm, N_G = 30, p_x = 13 mm, d_W = 50 mm <math>\phi_n = 14.5^{\circ}, \mu = 0.03$

(a) The pitch diameter of the gear is $p_t = p_x = 13 \text{ mm}$ $d_G = \frac{N_G p_t}{\pi} = \frac{30(13)}{\pi} = 124 \text{ mm}$ $C = \frac{d_G + d_W}{2} = \frac{50 + 124}{2} = 87 \text{ mm}$ $L = p_x N_W = 13(2) = 26 \text{ mm}$ From Eq. (13-28)

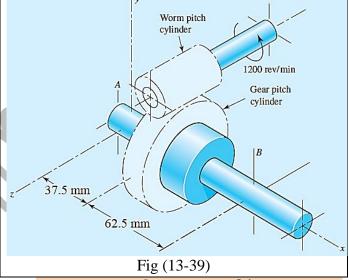


Fig (13-39)
$$\lambda = \tan^{-1} \frac{L}{\pi d_W} = \tan^{-1} \frac{26}{\pi (50)} = 9.40^{\circ}$$

$$| N_W | \frac{\pi d_W n_W}{60} = \frac{\pi (0.05)(1200)}{60} = 3.14 \frac{m}{s}$$

$$| N_G | \frac{n_G}{n_P} = \frac{N_P}{N_G} \rightarrow n_G = \left(\frac{2}{30}\right)(1200)$$

$$= 80 \text{ rpm}$$

$$V_G = \frac{\pi d_G n_G}{60} = \frac{\pi (0.124)(80)}{60} = 0.519 \frac{m}{s}$$

$$| N_G | N_G$$

حساب القوة المماسية للدودة من القدرة.

$$W_{Wt} = \frac{H}{V_W} = \frac{750}{3.14} = 239 \text{ N}$$

This force acts in the negative x direction, the same as in Figure 13–36. Using Figure 13–38, we find $\mu = 0.03$.

Then, the first equation of Equation (13–43) gives

$$W = \frac{W_x}{(\cos \phi_n \sin \lambda + \mu \cos \lambda)}$$

$$= \frac{239}{(\cos 14.5^\circ \sin 9.40^\circ + 0.03 \cos 9.40^\circ)}$$

$$= 1273 \text{ N}$$

$$W_y = W \sin \phi_n = 1270 \sin 14.5^\circ = 314 \text{ N}$$

$$W_z = W(\cos \phi_n \cos \lambda - \mu \sin \lambda) = 1273(\cos 14.5^\circ \cos 9.40^\circ - 0.03 \sin 9.40^\circ) = 1234 \text{ N}$$

We now identify the components acting on the gear as:

$$W_{Ga} = W_{Wt} = -W_x = 239 N$$

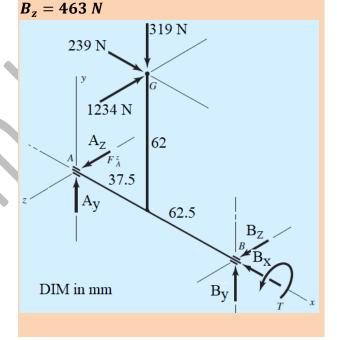
 $W_{Gr} = W_{Wr} = -W_y = 319 N$
 $W_{Gt} = W_{Wa} = -W_z = -1234 N$

لغرض تحديد اتجاهات القوى وردود افعالها باستخدام قاعدة اليد اليمنى لدوران الدودة يتم الإشارة بالإبهام الى المحور Z، ولتوضيح ذلك تخيل حركة البرغي والصامولة اذ يتم مشابهة حركة الدودة بحركة البرغي إذا كانا بنفس اتجاه الحلزنة، قم بتدوير البرغي بيدك اليمنى ممسكا الصامولة بيدك اليسرى، سوف تتحرك الصامولة باتجاه يدك اليمنى، من خلال هذا التشابه نستنتج ان السطح العلوي للترس سوف يتحرك باتجاه Z السالب، في هذه الحالة اذا نظرنا الى الترس

باتجاه x السالب نلاحظ انه يدور باتجاه عقرب الساعة حول المحور x.

$$B_x = W_{Ga} = 239 N$$

 $\Sigma M_{Bz} = 0$
 $319(62.5) - 239(215.4) - F_y = 0$
 $F_y = 50 N$
 $\Sigma M_{By} = 0$
 $100A_z - 1234(62.5) = 0$
 $A_z = 771 N$
 $\Sigma F_y = 0$
 $B_y - 319 + 50 = 0$
 $B_y = 269 N$
 $\Sigma F_z = 0$
 $A_z + B_z - 1234 = 0$
 $771 + B_z - 1234 = 0$



PROBLEMS

13–2 A 15-tooth spur pinion has a module of 3 mm and runs at a speed of 1600 rev/min. The driven

gear has 60 teeth. Find the speed of the driven gear, the circular pitch, and the theoretical center to center distance.

- **13–3** A spur gearset has a module of 6 mm and a velocity ratio of 4. The pinion has 16 teeth. Find the number of teeth on the driven gear, the pitch diameters, and the theoretical center-to-center distance.
- 13–7 A parallel helical gearset consists of a 19-tooth pinion driving a 57-tooth gear. The pinion has a left-hand helix angle of 30°, a normal pressure angle of 20°, and a normal module of 2.5 mm. Find:
 - (a) The normal, transverse, and axial circular pitches

DIM in MM

13-18

10-in dia.

142 →

- (b) The transverse pitch and the transverse pressure angle
- (c) The addendum, dedendum, and pitch diameter of each gear
- 13–15 A parallel-shaft gearset consists of an 18-tooth helical pinion driving a 32tooth gear. The pinion has a left-hand helix angle of 25°, a normal pressure angle of 20°, and a normal module of 3 mm. Find:
- (a) The normal, transverse, and axial circular pitches
- (b) The transverse module and the transverse pressure angle
 - (c) The pitch diameters of the two gears
- 13–16 The double-reduction helical gearset shown in the figure is driven through shaft a at a speed of 700 rev/min.

13–35 The figure shows the

dimensions for a 22 kW, 900 rev/min motor. The frame is

electric-motor

225

DIM in MM

frame

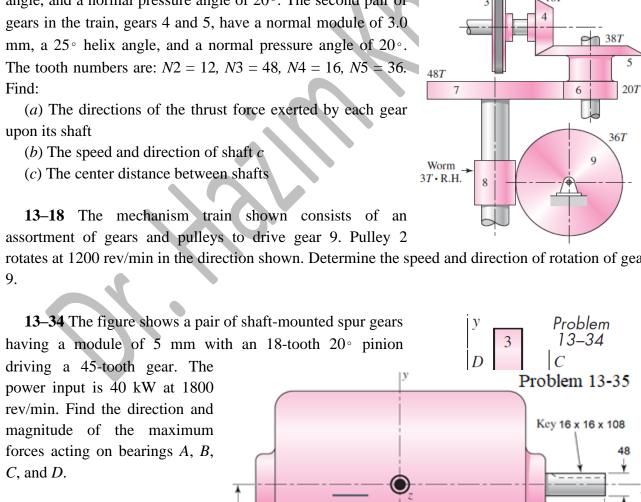
Gears 2 and 3 have a normal module of 2.0 mm, a 30° helix angle, and a normal pressure angle of 20°. The second pair of

13-18 The mechanism train shown consists of an rotates at 1200 rev/min in the direction shown. Determine the speed and direction of rotation of gear

285

390

290



bolted to its support using four 19 mm bolts spaced 285 mm apart in the view shown and 355 mm apart when viewed from the end of the motor. A 6 mm module, 20° spur pinion having 20 teeth and a face width of 52 mm is keyed to the motor shaft. This pinion drives another gear whose axis is in the same xz plane. Determine the maximum shear and tensile forces on the mounting bolts based on 200 percent overload torque. Does the direction of rotation matter?

13–36 Continue Prob. 13–24 by finding the following information, assuming a module of 4 mm.

- (a) Determine pitch diameters for each of the gears.
- (b) Determine the pitch line velocities (m/sec) for each set of gears.
- (c) Determine the magnitudes of the tangential, radial, and total forces transmitted between each

set of gears.

- (d) Determine the input torque.
- (e) Determine the output torque, neglecting frictional losses.
- 13–49 Gear 2, in the figure, has 16 teeth, a 20° transverse angle, a 15° helix angle, and a module of 4 mm. Gear 2 drives the idler on shaft b, which has 36 teeth. The driven gear on shaft c has 28 teeth. If the driver rotates at 1600 rev/min and transmits 6 kW, find the radial and thrust load on each shaft.
- 13–43 The figure shows a 16T 20° straight bevel pinion driving a 32T gear, and the location of the bearing centerlines. Pinion shaft a receives 1.9 kW at 240 rev/min. Determine the bearing reactions at A and B if A is to take both radial and thrust loads.

